Light Microscopy
Problem Set 4

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November 18, 2019

Please hand in the solutions of this problem set after the lecture on Decmber 2, 2019. It will be discussed in the seminar on December 9, 2019

1 Snells Law

Let us assume that two media with different refractive index n, e.g. air and water share the same, plane interface (Figure 2). In medium 1, a plane electromagnetic wave is incident. At the interface the incident wave is partially reflected and partially transmitted to medium 2.

For the incident, reflected and the transmitted wave following equations are given:

$$\mathbf{E} = \begin{cases} \mathbf{E_0} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + E_0'' \cdot e^{(i(\mathbf{k}''\cdot\mathbf{r}-\omega t)}, & z > 0 \\ \mathbf{E_0'} e^{i(\mathbf{k}'\cdot\mathbf{r}-\omega t)}, & z < 0 \end{cases} \quad (1)$$

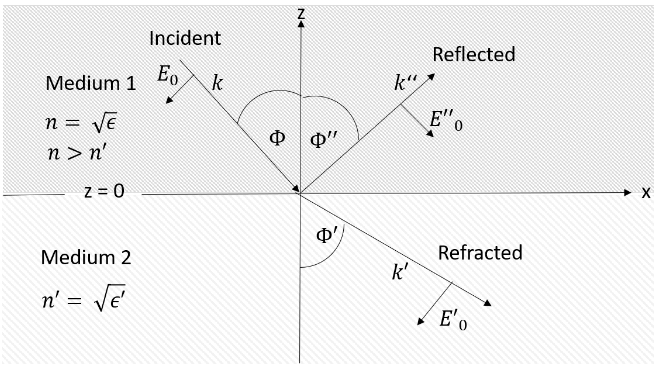

Figure 1

We assume for the material constants

$$\mu' = \mu = 1, n = \sqrt{\epsilon} = \text{real and } n' = \sqrt{\epsilon'} = n'_{real} + in'_{imaginary} \quad (2)$$

(a) The boundary conditions are given at the interface ($z = 0$) with

$$\mathbf{e_z} \cdot \mathbf{D},\ \mathbf{e_z} \cdot \mathbf{B},\ \mathbf{e_z} \times \mathbf{E},\ \mathbf{e_z} \times \mathbf{H} \text{ continuous at } z = 0 \quad (3)$$

Derive from the given information Snell's law. Hint: Which conditions for $\omega, \omega', \omega''$ and k, k', k'' must be fulfilled with the given boundary conditions?

(b) Consider now a real refraction index n and a complex n' with $n' = n'_r + i\kappa'$ with $\kappa` \ll n'_r$. Determine the complex angle ϕ', which can be calculated using Snell's law. Put therefore $\phi' = \alpha' + i\alpha''$ and calculate the real angles α' and α''.

Hint: Show that following statement is true:

$$sin(\phi') \approx sin(\alpha') + i\alpha'' cos(\alpha') \quad (4)$$

(c) From Snell's law follows for two transparent mediums (n, n' real) with $n > n'$

$$\phi \leq \phi_{TR} = \arcsin(n'/n) \ \ (n > n') \tag{5}$$

with ϕ_{TR} = angle of total reflection. Show that for $\phi > \phi_{TR}$ the z-component of the transmitted wave vector $\mathbf{k}'$ is purely imaginary. What does that mean for the wave in medium 2?

2 Lens as a Fourier Transformer

Let's take a look again to the FOMO Script Chapter 4.1: Imaging of arbitrary optical field with thin lens.

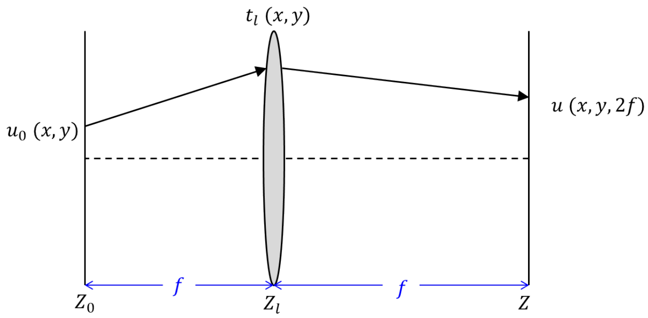

Figure 2: Geometry for optical propagation through a lens.

(a) Write down the transfer function or field transmission of a perfect thin lens. Explain all the variables you are using.

(b) Explain what the exponential term represents.

(c) Considering the field at plane Z_0 as $u(x, y)$, calculate the field distribution at the plane Z_l, $u_-(x, y, f)$. Hint: use the Fresnel diffraction integral.

(d) Calculate the field distribution after the interaction with the lens, $u_+(x, y, f)$.

(e) Now calculate the field distribution at Z.

3 Invariance requirements for similarity in imaging

The Abbe sine condition and the Herschel condition are invariance requirements that an optical system needs to fulfill, individually, to obtain an image similar to the object.

(a) Considering a constant transverse spatial frequency scaling, use the Fourier picture to derive a condition relating the angles of the k-vectors in object space to corresponding k-vectors in image space. Hence describe the obtained relation (Abbe sine condition) and its impact on the image.

(b) Derive a condition relating half of the previous mentioned angles to obtain a constant longitudinal spatial frequency magnification (Herschel condition).

(c) Under which condition can the Herschel and the Abbe sine condition both be fulfilled simultaneously? Which optical system fulfills these requirements?.

All lecture scripts and additional material will be posted at: https://nanoimaging.de/teaching/current-semester/