Light Microscopy
Problem Set 1

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Please hand in the solutions of this problem set after the lecture on October 21, 2019. It will be discussed in the seminar on October 28, 2019

1 Symmetry properties of Fourier-Transformations

Consider f as a sufficiently regular function of $\mathbf{r} \in \mathbb{R}^n$ and c as a scaling factor with $c = \frac{1}{(2\pi)^{\frac{n}{2}}}$. Then the Fourier-Transform can be written as:

$$\mathcal{F}\{f(\mathbf{r})\}(\mathbf{k}) \equiv F(\mathbf{k}) = c \int_{\mathbb{R}^n} f(\mathbf{r})\, e^{-\mathbf{ik}\cdot\mathbf{r}}\, \mathrm{d}\mathbf{r} \tag{1}$$

$$= c \int_{\mathbb{R}^n} \cos(2\pi\mathbf{k}\cdot\mathbf{r})\, \mathrm{d}\mathbf{r} - ic \int_{\mathbb{R}^n} f(\mathbf{r}) \sin(2\pi\mathbf{k}\cdot\mathbf{r})\, \mathrm{d}\mathbf{r} \tag{2}$$

Obviously the first integral is an even function and the second integral an uneven function of f.

Which properties has $F(\mathbf{k})$ if f is:

(a) even
(b) odd
(c) even and real-valued
(d) odd and real-valued
(e) even and imaginary-valued
(f) odd and imaginary-valued
(g) hermitian, $(f(\mathbf{r}))^* = f(-\mathbf{r})$
(h) anti-hermitian, $(f(\mathbf{r}))^* = -f(\mathbf{r})$
(i) real-valued
(j) pure imaginary-valued.

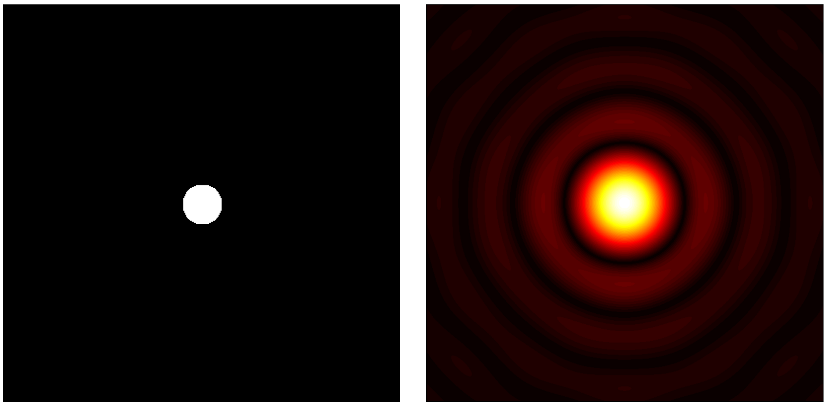
2 Intensity density spectra

Take your FOMO Script Chapter 3.3: Fraunhofer diffraction at plane masks (paraxial).

a) Explain with your own words the conditions to obtain a Fraunhofer diffraction pattern and how it is related to the Fourier transformation of the mask.

b) Calculate the 2D diffraction patterns of the following apertures. Consider uniform plane wave illumination. Indicate the correspondent dimensions of the patterns with respect to the indicated geometry variables.

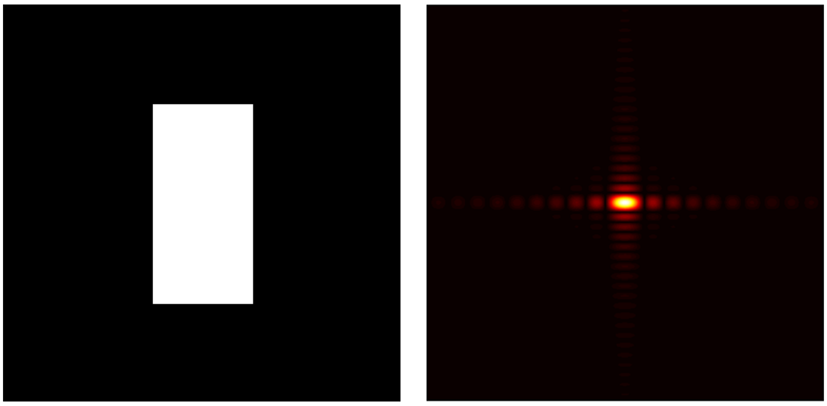
i) Circular mask



(a) (b)

Figure 1: Fraunhofer diffraction of a (a) circular aperture of radius r.

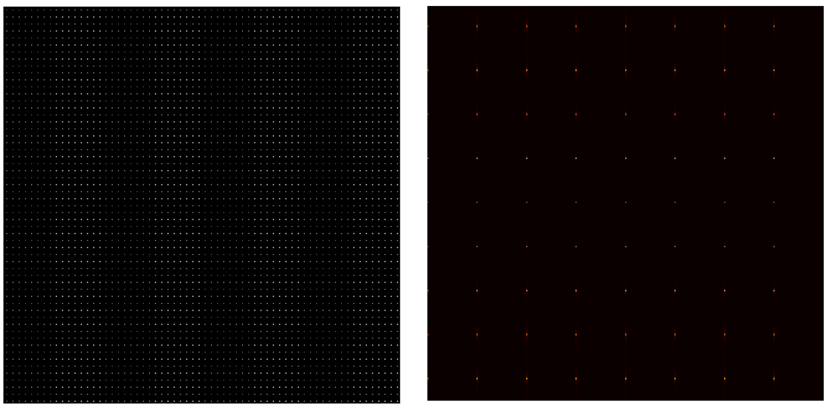
ii) Rectangular mask



(a) (b)

Figure 2: Fraunhofer diffraction of a rectangular aperture of dimensions a x b.

c) Now indicate the relation between a (the lattice spacing of Figure 3a) and d, the Fraunhofer diffraction pattern lattice spacing (Figure 3b).



(a) (b)

Figure 3: Fraunhofer diffraction of a lattice.