Light Microscopy
Problem Set 2

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Please hand in the solutions of this problem set after the lecture on November 4, 2019. It will be discussed in the seminar on November 11, 2019

1 Convolution Theorem

Given $\mathcal{F}\{f(x)\}(k) = F(k)$ and $\mathcal{F}\{g(x)\}(k) = G(k)$. Their Convolution is defined by

$$F(k) \otimes G(k) = \int_{-\infty}^{\infty} F(\xi) \cdot G(k - \xi) \mathrm{d}\xi \tag{1}$$

Using this definition, proof the equality:

$$\mathcal{F}\{f(x) \cdot g(x)\} = F(k) \otimes G(k) \tag{2}$$

2 Transforming Differential Equations

Show that, in Fourier-Space, the two-dimentional Laplace Operator $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is proportional to a multiplication with the quadratic function of the wave-vector k according to:

$$\mathcal{F}\{\nabla^2 g(x,y)\} \propto (k_x^2 + k_y^2)\mathcal{F}\{g(x,y)\} \tag{3}$$

3 Picturing some 2D FT properties

Match the following uniform illuminated apertures (Figure 4) with their correspondent Fraunhofer diffraction patterns (Figure 5). Explain by naming the theorems you apply or other considered criteria.

Hint: Some of the following apertures can be described in terms of the masks discussed in the Homework Set 1. You may think about subtractions, convolutions, symmetries...

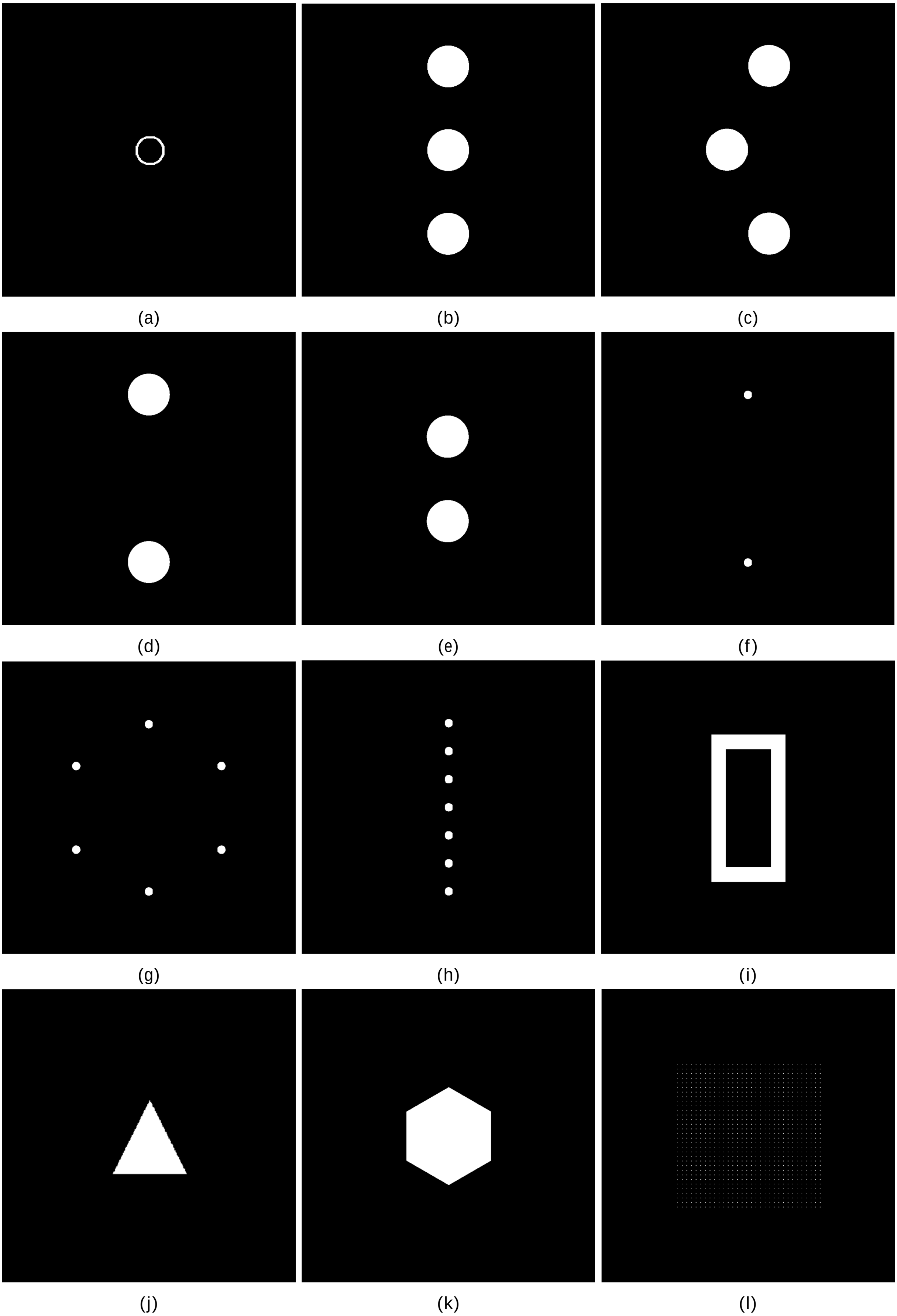

Figure 1: Masks.