

# Microscopy

## Problem Set 4

May 27, 2019

Please hand in the solutions of this problem set next lecture on May 31st of 2019.

### 5 Image formation in ray optics

- Draw a 4f system with lenses of diameter  $d$  focal lengths  $f_1$  and  $f_2$ . Calculate the magnification and the aperture angle  $\alpha$ .
- Draw the chief and marginal ray coming from an off axis point in the object plane. Explain with your own words how the image is formed in this case.
- Name the optical effects that cannot be described with ray optics.

### 6 Image formation in wave optics

- With the Huygens principle, explain interference and diffraction.
- Explain and illustrate the following experiments and therefore the concepts of coherence:
  - Michelson Interferometer, Temporal coherence.
  - Young's double slit experiment, Spatial coherence. Draw the obtained pattern (dependent of  $d$ , the distance between slits) and explain the origin of the maxima and minima.

### 7 Fourier transformations

- Read the paper attached and explain how an object can be described as a superposition of sine or cosine functions.
- With the previous concept, explain what is a Fourier transform.
- With the previous concept, describe the process of image formation shown in the animation "Towards the image: Constructing from waves" (Lecture 4).

## 8 Fluorescence

- a) What is the most important property of fluorescence and why?
- b) What laser intensity  $I$  (in  $\text{W}/\text{cm}^2$ ) of  $\lambda = 532 \text{ nm}$  has to be employed to excite a fluorophore with an absorption cross-section  $\epsilon =$  with a rate  $k = ?$  (Hint:  $k = \sigma \times$  number of photons, with molecular absorption cross-section  $\sigma$  for a single photon).
- c) The energy difference to the first excited vibronic state of the electronic ground state of a fluorophore is  $\Delta E = 750 \text{ cm}^{-1}$ . How much has the fluorophore have to be cooled down to reduce the population of the first excited vibronic state to a minimum ( $< 0.01$ )? (Hint: The relative population of higher vibronic states is given by the Boltzmann distribution).
- d) A fluorophore has a lifetime of  $\tau = 4.0 \text{ ns}$ . An interaction introduces an additional de-excitation process with rate constant  $k =$ . What is the change in quantum yield (before 1) and lifetime?
- e) (i) Two fluorophores have a lifetime of  $\tau = 4.0 \text{ ns}$  and  $1.0 \text{ ns}$ . What is their respective change in anisotropy  $r$  when the hydrodynamic radius  $R_{rot}$  of the investigated (spherical) molecule (rotating at a temperature  $T = 25 \text{ C}^\circ$ ) in a solvent with viscosity  $\eta = 1 \text{ cP}$ ,  $r_0 = 0.2$ ) changes from  $r_m = 1 \text{ nm}$  to  $3 \text{ nm}$ ? (Hint:  $1/r = 1/r_0(1 + \tau/\tau_c)$  with rotational correlation time  $\tau_c = 1/(6D_r)$  and rotational diffusion coefficient  $D_r = k_B T / (8\pi\eta R_{rot}^3)$ ).
- (ii) In which case is it possible to monitor the rotational decay  $r(t) = r_0 \exp(-t/\tau_c)$ ? (Hint: The total fluorescence intensity decays with the lifetime).
- f) (i) How long does it take to image a  $30 \mu\text{m} \times 100 \mu\text{m} \times 100 \mu\text{m}$  large area with widefield (observation spot diameter  $> 100 \mu\text{m}$ , camera frame rate  $100 \text{ Hz}$ , axial step size  $1.5 \mu\text{m}$ ), confocal (focal spot size  $0.3 \mu\text{m}$  laterally and  $1 \mu\text{m}$  axially, scan step size  $0.2 \mu\text{m}$  laterally and  $0.5 \mu\text{m}$  axially, scan dwell time  $1 \mu\text{s}$ ) and lightsheet (sheet size  $> 100 \mu\text{m}$  (lateral)  $\times > 100 \mu\text{m}$  (lateral)  $\times > 3 \mu\text{m}$  (axial), with sheet scan step size  $1.5 \mu\text{m}$  and camera frame rate  $100 \text{ Hz}$ ).
- (ii) Which method will give the best contrast and why?