Light Microscopy
Problem Set 1

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Please solve this Problem-Set until Monday the 23th of October 2017. You will present it in the seminar.

1 Symmetry properties of Fourier-Transformations

Consider f as a sufficiently regular function of x $\in \mathbb{R}$. Then the Fourier-Transform can be written as:

$$\mathcal{F}\{f(x)\}(k) \equiv F(k) = \int_{-\infty}^{\infty} f(x)\cos(2\pi kx)\mathrm{d}x - i\int_{-\infty}^{\infty} f(x)\sin(2\pi kx)\mathrm{d}x \tag{1}$$

Obviously the first integral is an even function and the second integral an uneven function of f.

Which properties has $F(k)$ if f is:

(a) even
(b) odd
(c) even and real-valued
(d) odd and real-valued
(e) even and imaginary-valued
(f) odd and imaginary-valued
(g) hermitian, $(f(x))^* = f(-x)$
(h) anti-hermitian, $(f(x))^* = -f(-x)$
(i) real-valued
(j) pure imaginary-valued.

2 A first Fourier-Transformation

Calculate the Fourier-Transformation of the following function:

$$\mathrm{rect}(x) = \begin{cases} 1, & |x| \leq 1/2 \\ 0, & |x| > 1/2 \end{cases} \tag{2}$$

where rect is the standard rectangular function.

3 Convolution Theorem

Given $\mathcal{F}\{f(x)\}(k) = F(k)$ and $\mathcal{F}\{g(x)\}(k) = G(k)$. Their Convolution is defined by

$$F(k) \otimes G(k) = \int_{-\infty}^{\infty} F(\xi) \cdot G(k-\xi)\mathrm{d}\xi \tag{3}$$

Using this definition, proof the equality:

$$\mathcal{F}\{f(x) \cdot g(x)\} = F(k) \otimes G(k) \tag{4}$$

4 Transforming Differential Equations

Show that, in Fourier-Space, the two-dimentional Laplace Operator $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is proportional to a multiplication with the quadratic function of the wave-vector k according to:

$$\mathcal{F}\{\nabla^2 g(x,y)\} \propto (k_x^2 + k_y^2)\mathcal{F}\{g(x,y)\} \tag{5}$$

All lecture scripts and additional material will be posted at:
https://nanoimaging.de/teaching/current-semester/light-microscopy-201718/